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Third Term Test - Grade 13 - 2025

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II

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Combined Mathematics II

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Three Hours

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Additional Reading Time - 10 minutes

**Use additional reading time to go through the question paper, select the questions you will answer
decide which of them you will prioritise.**

Index number

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Instructions:

- * *This question paper consists of two parts
Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17)*
- * **Part A:**
*Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- * **Part B:**
*Answer **five** questions only. Write your answers on the sheets provided*
- * *At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on the top of **Part B** and hand them over to the supervisor*
- * *You are permitted to remove **only Part B** of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
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	10	
B	11	
	12	
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	17	

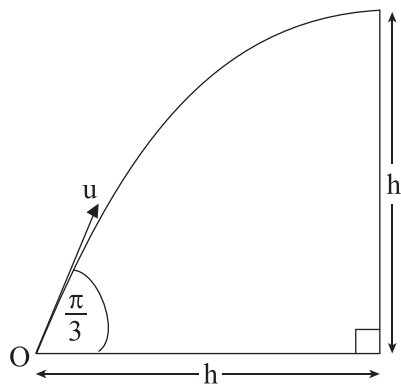
Total

In numbers	
In words	

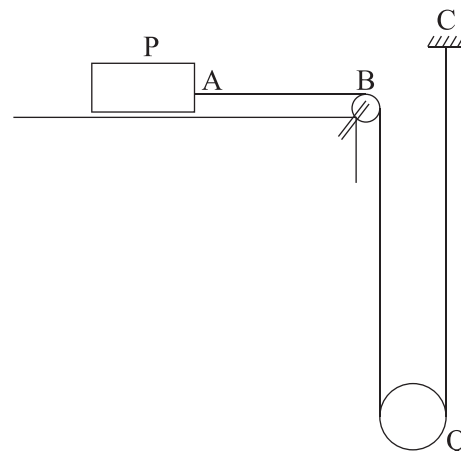
Part A

01. Particle P of mass $3m$ and particle Q of mass m moving towards each other with velocities $2u$ and $3u$ respectively, along a straight line on a smooth horizontal table collide directly. The coefficient of restitution between particles P and Q is e . If the particles move in opposite directions after the collision, show that $e > \frac{3}{5}$.

02. A particle is projected from a point O on a horizontal floor with initial velocity u and at an angle $\frac{\pi}{3}$ to the horizontal. The particle just clears a vertical wall of height h located at a horizontal distance h from O, as shown in the figure. Show that $u^2 = \frac{2gh}{\sqrt{3}-1}$.
-
- The diagram illustrates a projectile motion problem. A particle is launched from point O on a horizontal floor. The launch angle is $\frac{\pi}{3}$ relative to the horizontal axis. The particle's trajectory is a parabolic arc that reaches its maximum height exactly above a vertical wall. This wall has a height of h and is positioned at a horizontal distance of h from the origin O. The particle just clears the top of the wall. A right-angle symbol is shown at the base of the wall where it meets the ground.

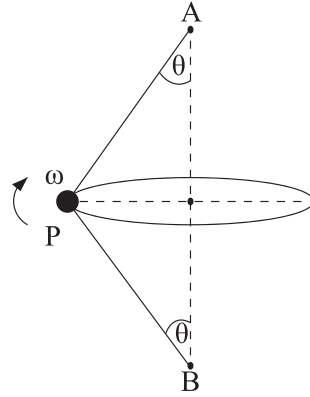


03. An object P of mass $3m$ is placed on a rough horizontal plane. One end of a light inextensible string passing over a smooth pulley B fixed at the edge of the plane and passing underneath a smooth movable pulley Q of mass $5m$ is connected to the object P and the other end of the string is connected to a fixed point C as shown in the figure. The part AB is horizontal and the other parts of the string are vertical. The coefficient of the friction between P and the plane is $\frac{1}{3}$. The system is released from rest when the string is taut. Obtain equations sufficient to determine the tension of the string.



04. A car of mass M kg moves up along a straight road inclined at an angle θ ($0 < \theta < \frac{\pi}{2}$) to the horizontal, with the engine working at constant power H kW. If the resistance to the motion is R N, find the velocity of the car. Next, the car moves along a straight horizontal road against the same resistance R N with the same power H kW. Find the acceleration of the car when its velocity is U ms⁻¹.

05. A particle P of mass m is connected to two fixed points A and B on a same vertical line by two light inextensible strings, each of length l . The particle P moves in a horizontal circle with constant angular velocity ω making angle θ with the vertical ($\theta < \theta < \frac{\pi}{2}$). If both strings are taut, show that



$$\omega > \sqrt{\frac{g \sec \theta}{\ell}}$$

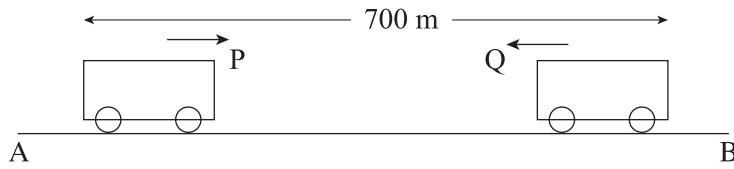
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06. Let $\underline{p}$ and $\underline{q}$ are two unit vectors such that $\underline{p} \cdot \underline{q} = 1$. Given that $\underline{a} = m\underline{p} + \underline{q}$ and $\underline{b} = 2\underline{p} + n\underline{q}$ where $m, n \in \mathbb{R}$. If $\underline{a}$ and $\underline{b}$ are perpendicular to each other and $\underline{a} - \underline{b}$ is parallel to $\underline{q}$, find the values of m and n .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Part B

★ Answer only **five** questions



11. (a) A and B are two points located at distance 700 m on a straight road. At time $t = 0$, a car P starts from rest at A in the direction $\overrightarrow{AB}$ and obtains velocity of $4f\text{ ms}^{-1}$ within 10 seconds moving with constant acceleration. Then P moves with this constant velocity for another 10 seconds. At time $t=20\text{ s}$, P starts to move with constant retardation and comes to the rest at C within 5 seconds. After resting 5 seconds at C, P starts from rest and moves in the same direction with constant acceleration for t_0 seconds and obtains the velocity $4f\text{ ms}^{-1}$.

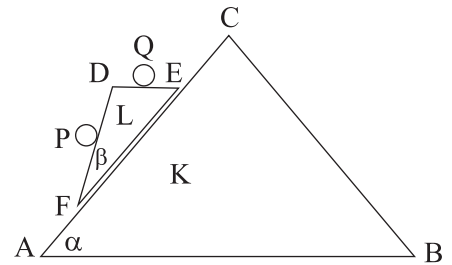
At $t = 20\text{ s}$, a motor car B starts from the velocity $2f\text{ ms}^{-1}$ in the direction $\overrightarrow{BA}$ and obtains the velocity of $4f\text{ ms}^{-1}$ within 10 s with constant acceleration. When the car P starts from C, the distance between P and Q is 200 m . When the car B obtains the velocity $6f\text{ ms}^{-1}$ from $4f\text{ ms}^{-1}$ within t_0 seconds, P and Q meet each other.

Sketch velocity time graphs for the motion of P and Q in the same diagram.

Hence or otherwise show that

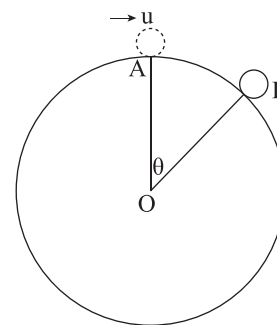
- (i) $f = 5\text{ ms}^{-2}$
(ii) $t_0 = \frac{40}{7}\text{ s}$
- (b) A boat observes a ship at a distance $a\text{ km}$ at an angle 60° north of west. The ship is sailing due east with uniform speed $u\text{ kmh}^{-1}$. The boat moves with velocity $v\text{ kmh}^{-1}$ to meet the ship. Here $\frac{\sqrt{3}}{2}u < v < u$.
By considering the motion of the boat relative to the ship, show that there are two possible directions for the boat to reach the ship. Show that the difference of times taken by the boat using these two paths is $\frac{a\sqrt{4v^2 - 3u^2}}{u^2 - v^2}$ hours.

12. (a) The figure shows the vertical cross-section through the center of mass of two smooth uniform wedges K and L and two particles P and Q. AC, DE, DF and EF are lines of greatest slope of the faces containing them. $\hat{BAC} = \alpha$ and $\hat{DFE} = \beta$. The wedge K of mass M_1 is placed on a smooth horizontal table such that the face containing AB touching the table. The wedge L of mass M_2 is placed on AC of wedge K, such that the face containing EF of wedge L touching AC. A particle P of mass m_1 is placed on DF and the particle Q of mass m_2 is placed on DE. Here DE is horizontal. When the system is released from rest, the wedge L moves on AC, the particle P moves on DF and the particle Q moves on DE. Write down the equations sufficient to determine the acceleration of the wedge K.



- (b) O is the center of a sphere of radius a containing smooth surface. A particle P of mass m is placed at the highest point A of the sphere and a horizontal velocity u is given to the particle. Show that the velocity of the particle P when OP makes an angle θ with the vertical is given by $v^2 = u^2 + 2ga(1 - \cos\theta)$

If $u = \sqrt{\frac{ga}{2}}$, show that $\theta = \cos^{-1}(\frac{5}{6})$ when the particle P leaves the sphere.



13. One end of a light elastic string of natural length l is tied to a fixed point O and the other end is attached to a particle P of mass m . The particle P is in equilibrium at A such that $OA = 3l$. Show that the modulus of elasticity of the string is $\frac{mg}{2}$

Then the particle is projected down wards with a velocity of $\sqrt{3gl}$ from A. Show that the equation of the motion of the particle P is given by

$$\ddot{x} + \omega^2(x - 3l) = 0$$

Here $x > l$ for $OP = x$ and $\omega(> 0)$ is a constant to be determined. Rewrite the equation of the motion by taking $X = x - 3l$. Find the center and the period of the simple harmonic motion of P.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find C . Here C is the amplitude of the motion. Show that the distance between O and the lowest point of the motion is $(3\sqrt{6})l$.

Also show that the particle P does not reach the point O again.

Further show that the time taken by the particle P to reach the highest point of the motion from A is

$$\sqrt{\frac{2l}{g}} \left[\frac{3\pi}{2} - \cos^{-1}\left(\frac{\sqrt{2}}{3}\right) + \frac{1}{\sqrt{2}} \right]$$

14. (a) In the usual notation, the position vectors of the points A, B and C are given by $\underline{a} = \underline{i} + \underline{j}$, $\underline{b} = 2\underline{i} + 4\underline{j}$ and $\underline{c} = 3\underline{i} + 2\underline{j}$ respectively. P is a point on AB such that $\overrightarrow{AP} = \lambda \overrightarrow{AB}$. Show that the position vector of P is $\underline{p} = (1 + \lambda)\underline{i} + (1 + 3\lambda)\underline{j}$. Here $\lambda \in \mathbb{R}$.

Let P be the foot of the perpendicular drawn from C to AB. Find the value of λ and show that $\underline{p} = \frac{3}{2}\underline{i} + \frac{5}{2}\underline{j}$

D is a point on CP produced such that CB is parallel to AD. Find the position vector of D in terms of $\underline{i}$ and $\underline{j}$. By considering $\overrightarrow{AD} \cdot \overrightarrow{PD}$, find $\hat{ADP}$

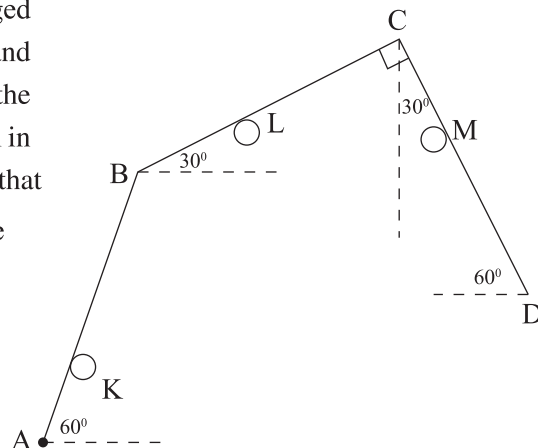
- (b) O $\equiv$ (0, 0), A $\equiv$ (9, 0), B $\equiv$ (9, 7) and C $\equiv$ (0, 7) are vertices of rectangle OABC on a OXY coordinate plane. Four forces P, Q, R and S act along OA, OC, BA and BC respectively in the directions indicated by the order of the letters. It is given that the resultant force of the system act along the line $4x + 3y = 12$ and the moment about O in anticlockwise is 12 Nm .

If $P = 9$, find the values of Q, R and S. Here distances are given in meters. Hence find the magnitude of the resultant.

Then, a couple is introduced to the system such that the line of action of resultant of the new system, passes through A. Find the moment of the couple introduced.

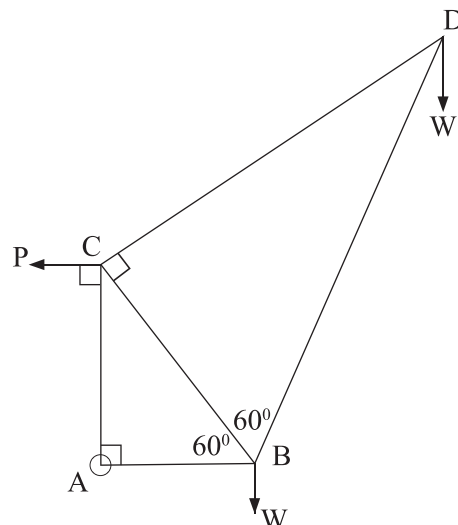
15. (a) Three uniform rods AB, BC and CD each of weight W and length $6a$ are smoothly jointed at B and C. The end A is hinged to fixed point. $AK = BL = CM = 2a$ and the rods AB, BC and CD make angles 60° , 30° and 60° respectively with the horizontal. $\angle BCD = 90^\circ$. The framework is kept in equilibrium in a vertical plane by three smooth pegs at K, L and M. Show that

- the magnitude of the reaction made by the peg M on the rod CD is $\frac{3W}{4}$
- the magnitude of the reaction made by the peg L on the rod BC is $\frac{9\sqrt{3}W}{4}$
- Also find the horizontal and the vertical components of reaction at the joint B.



- (b) A framework of five rigid light rods AB, BC, AC, CD and BD freely jointed at their ends as shown in the figure. Given that $AB = a$, $\angle BAC = 90^\circ$, $\angle BCD = 90^\circ$, $\angle ABC = \angle CBD = 60^\circ$. Two weights each of W are hung at the joints B and D. The framework is smoothly hinged at A to a fixed point and kept in equilibrium with AB horizontal by a horizontal force P applied at C.

- Find the value of P .
- Draw stress diagrams for the joints C, B and D on the same figure, using Bow's notation. Hence find the stresses in the rods stating whether they are tensions or thrusts.



16. Show that the center of mass of

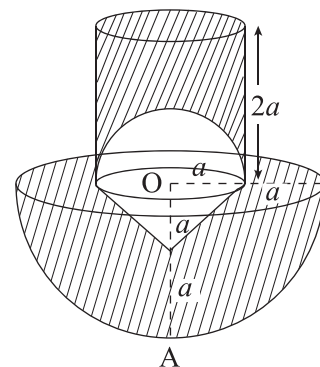
- a uniform solid hemisphere of radius a is at a distance $\frac{3a}{8}$ from the center.
- a uniform solid right circular cone of height h is at a distance $\frac{1}{4}h$ from the center at the base.

A solid cone of radius a and height a is removed from a solid hemisphere of radius $2a$ and center O. The density of the hemisphere is ρ .

A solid hemisphere of radius a is removed from a solid cylinder of radius a and height $2a$ made with a material of density $k\rho$.

The remaining parts of the solid hemisphere and the solid cylinder are fixed rigidly as shown in the figure so as to form a composite body S. Show that the center of mass of the composite body S lies at a distance $\frac{(73 + 53k)a}{4(15 + 4k)}$ from A.

If the center of mass of S lies at O, find the value of k .



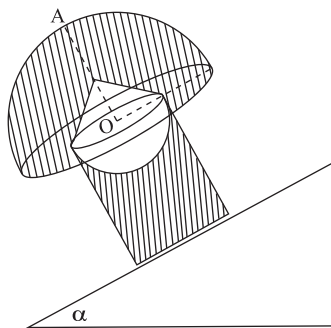
Assume that k is above value.

Then, the composite body S is placed on a rough plane inclined at an angle α to the horizontal as shown in the figure.

The coefficient of friction between the plane and the body is $\frac{1}{3}$. If the composite body S does not slide along the plane show that $\tan \alpha \leq \frac{1}{3}$.

Then a gradually increasing force P is applied at the point A on the surface of the body S in the upward direction parallel to the inclined plane. If the weight of the body S is W , show that

$P = \frac{W}{4} (\cos \alpha + 2 \sin \alpha)$ when the body is about to topple.



17. (a) The box A contains two red balls and 4 blue balls which are identical in all aspects except colour. The box B contains a red ball identical to above balls. Three balls are taken out at random from the box A and put them in to the box B. Then a ball is taken out from the box B.

- Find the probability that the ball taken out from the box B is red.
- Given that the ball taken out from the box B is red. Find the probability of putting one red ball and two blue balls to the box B from box A.

- (b) Over time allowances paid for 40 employees in a certain day are coded by subtracting 1000 from each allowance and then dividing the remainder by 10.

The frequency distribution of coded allowances is given below.

Coded Allowances (in rupees)	Frequency
1 - 3	4
3 - 5	a
5 - 7	15
7 - 9	b
9 - 11	5

Given that the mode of the coded allowances is $\frac{43}{7}$.

Show that $a = 7$ and $b = 9$.

Estimate the mean and the standard deviation of the coded allowances.

Then find the mode, mean and the standard deviation of actual allowances.